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Inflation Dynamics in the U.S.: Global but Not Local Mean Reversion

A stylized fact of U.S. inflation dynamics is one of extreme persistence and possible unit root behavior. If so, the implications for macroeconomics and monetary policy are somewhat unpalatable. Our econometric analysis proposes a parsimonious univariate representation of the inflation process for the last 60 years, the nonlinear exponential smooth autoregressive. The empirical results confirm a number of the key features such as global stationarity, local unit root behavior, and lower persistence in the post-1983 period than in the pre-1983 period. We compare the forecasting ability of our model with that of competing univariate models and find that the nonlinear model outperforms the linear autoregressive model in the pre-1983 period and the random walk in the post-1983 period at short horizons.

JEL codes: C15, C22, E31

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A STYLIZED FACT of the dynamics of U.S. inflation, as first highlighted in the pioneering contribution of Nelson and Schwert (1977), clearly indicates that it is a very persistent process. In fact, many studies suggest that U.S. inflation contains a unit root (see, e.g., Barsky 1987, Brunner and Hess 1993). Moreover, the unit root property appears to be shared for a wide array of economies examined in O'Reilly and Whelan (2005) and Cecchetti et al. (2007). More recently, in influential contributions, Stock and Watson (2007) and Cogley and Sargent (2007) have

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parsimoniously modeled inflation as an unobserved component trend-cycle model with stochastic volatility, a model that in its reduced form also exhibits a unit root.

The unit root feature of inflation is now reflected in theoretical models of the inflationary process. Woodford (2006) allows for the unit root feature by assuming that the inflation target follows a random walk. Cogley and Sbordone (2006) reformulate the New Keynesian supply curve because the standard formulation is based on the assumption that inflation is stationary.¹ There are, however, severe economic and statistical problems with the assumption of a unit root in the inflation process. For instance, the assumption would imply, *ceteris paribus*, that the nominal exchange rate, via PPP, is an I(2) process. Moreover, asset arbitrage would require nominal asset returns in general to exhibit I(1) behavior, and this is dramatically at odds with empirical findings. Further, the assumption of a random walk in the inflation target in theoretical models implies that the target will ultimately take negative values. A negative inflation target is implausible in view of the zero lower bound on nominal interest.

The focus of this paper is to consider an alternative parameterization of the inflation process.² Recent monetary policy analyses argue that the central bank pursues an implicit or explicit inflation target and that adjustment to this target is nonlinear.³ One model of the policymaker that implies this reduced form behavior of the inflation rate is the opportunistic approach to disinflation set out by Orphanides and Wilcox (2002) and Aksoy et al. (2006). The key feature of their model, as stated by Aksoy et al., is that “a central bank controls inflation aggressively when inflation is far from its target, but concentrates on output stabilization when inflation is close to its target, allowing supply shocks and unforeseen fluctuations in aggregate demand to move inflation within a certain band.” In this regard it is relevant that Martin and Milas (2007) estimate threshold Taylor rules for the period 1983Q1–2004Q4 for the U.S. that are consistent with the opportunistic model. They suggest that the response of interest rates to inflation is zero when inflation is in the “band.” They also point out that the opportunistic approach to inflation has similarities with “constrained discretion” as advocated by Bernanke and Mishkin (1997) and Bernanke (2003).⁴

Another model that is consistent with inflation following a nonlinear process is that of the risk management of monetary policy (Kilian and Manganello 2007, 2008). In this model, the central bank reacts depending on where the predictive distribution of inflation lies to given upper and lower bounds such that it balances the risks of

1. As is well recognized, and discussed robustly in Cochrane (2007), there are related issues of indeterminacy in this literature.

2. Within the linear framework adopted in the extant literature, an alternative avenue is to consider whether inflation is fractionally integrated (see, e.g., Baillie, Chung, and Tieslau 1996). A major shortcoming of this literature, however, is that it does not allow for possible structural breaks in the series to reflect regime changes as reflected in the analyses of the U.S. Great Moderation. Regime changes are known to spuriously induce the fractional property (see, e.g., Diebold and Inoue 2001).

3. However, Gregoriou and Kontonikas (2009) model the first difference of the deviations of inflation rates from target as ESTAR process that is inconsistent.

4. The monetary approach to the stagflation of the 1970s and 1980s developed by Barsky and Kilian (2002) also incorporates a central bank reacting to inflation (or deflation) when it is above or below certain bounds. This model would also be consistent with the nonlinear dynamics discussed in our model later.

price instability and unsustainable economic growth. Kilian and Manganelli (2007) provide empirical support for this model and are able to reject the null hypothesis of quadratic preferences in the loss function, including rejection of quadratic preferences in inflation. These results imply that the Fed does not simply respond linearly to the conditional mean of inflation, but rather it allows it to fluctuate within a band of “price stability” whose bounds are not precisely disclosed.

Following from the discussion in the last two paragraphs, we conjecture that inflation behaves as a near unit root process for inflation rates close to the implicit target of the policymaker but is mean reverting for large deviations. A natural counterpart in exchange rate analysis is that transactions costs or the sunk costs of international arbitrage induce a nonlinear adjustment of the real exchange rate to purchasing power parity (PPP) (see Dumas 1992). Essentially, small deviations from PPP are left uncorrected if they are not large enough to cover transactions costs or the “sunk costs of international arbitrage.” Empirical work shows that the exponential smooth autoregressive (ESTAR) model provides a parsimonious fit to PPP data (see Kilian and Taylor 2003, Michael, Nobay, and Peel 1997, Paya and Peel 2006). In this respect, the nature of the implied inflation adjustment process is similar to that suggested to explain deviations from PPP.

One simple ESTAR process that captures the PPP dynamics and also the inflation adjustment mechanism postulated earlier can be represented as follows:

$$y_t = \alpha + e^{-\gamma(y_{t-d}-\alpha)^2} \left[\sum_{i=1}^p \beta_i (y_{t-i} - \alpha) \right] + u_t, \quad (1)$$

where y_t is the inflation rate, α is a constant, $\Phi(p) = \sum_{i=1}^p \beta_i$, u_t is a random disturbance term, and the transition function is $G(\cdot; \gamma) = e^{-\gamma(y_{t-d}-\alpha)^2}$, with $\gamma > 0$, and $d \geq 1$ is the delay parameter of variable y_t . Within this framework, the equilibrium is given by α . The ESTAR transition function is symmetric about $y_{t-d} - \alpha$. The parameter γ is the transition speed of the function $G(\cdot)$ toward 0 (or 1) as the absolute deviation grows larger or smaller. Particular emphasis is reserved for the unit root case, $\Phi(p) = 1$. In this case, y_t behaves as a random walk process when it is near α . When the deviations from equilibrium are larger, the magnitude of such deviations along with the magnitude of γ imply that $G(\cdot)$ is less than 1 so that y_t is mean reverting. This ESTAR model provides an explanation of why inflation deviations or PPP deviations analyzed from a linear perspective might appear to be described by either a nonstationary integrated I(1) process, or alternatively, described by fractional processes.⁵

The remainder of the paper is structured as follows. In the next section, we discuss and carry out a sequence of econometric tests to discriminate between the linear unit root or linear stationary models of inflation and the ESTAR model outlined earlier. We subsequently estimate the nonlinear model and undertake an analysis of the impulse

5. Pippenger and Goering (1993) show that the Dickey–Fuller tests have low power against data simulated from an ESTAR model. Michael, Nobay, and Peel (1997) illustrate that data that are generated from an ESTAR process can appear to exhibit the fractional property.

response functions for the preferred specification. Section 2 compares the forecasting performance of the nonlinear model relative to three univariate alternatives, namely, a random walk, a linear AR process, and an IMA/UC model. Concluding comments are offered in the last section.

1. NONLINEAR MODEL

1.1 Data and Linearity Testing

We examine quarterly U.S. inflation measured by the log difference of PCE chain-type index or GDP price index over the period 1947Q1 to 2004Q4. The data are available from the Federal Reserve Economic Database (FRED) and are seasonally adjusted.⁶ We divide the sample into two main subperiods for detailed analysis. These periods are 1947Q1 to 1982Q4 and 1983Q1 to 2004Q4, respectively. The second period corresponds to a dramatic reduction in the volatility of inflation following the Volcker deflation and is regarded as a different policy regime as demonstrated in the estimates of Taylor rules (see, e.g., Clarida, Galí, and Gertler 2000, Dolado, Dolores, and Ruge-Murcia 2004, Martin and Milas 2007). There is more debate about the precise beginning and ending of the first regime but the results are robust for the first sample and marginally more significant for the PCE index. Cogley and Sargent (2007) note colleagues in the Federal Reserve pay more attention to this measure of inflation for policy purposes. Consequently, we report analysis of the PCE index.

Within the framework we consider, the key empirical issue is that of discriminating between alternative specifications, so as to choose the most parsimonious statistical representation of inflation. To do so, we proceed by applying a number of recently developed linearity and unit root tests described in Appendix A. Overall, our battery of tests clearly suggests that a linear process, either stationary or nonstationary, can be rejected in favor of a nonlinear ESTAR process.⁷

1.2 Nonlinear Estimates: The ESTAR Model

In Tables 1a and 1b, we present the results of the estimation of ESTAR models using nonlinear least squares for the main subperiods, as justified earlier, and a few other periods for comparison of parameter stability.⁸ The ESTAR model in the first period is jointly estimated with a GARCH(1, 1) process. The estimated coefficients in

6. These data were kindly made available to us by Timothy Cogley can be found at <http://research.stlouisfed.org/fred2/>. The series have FRED mnemonics PCECTPI and GDPCTPI, respectively. See Paya, Duarte, and Holden (2007) for a discussion of prices sampled at different frequencies and implications on inflation persistence.

7. These results are also in contrast to those found in Pivetta and Reis (2007) where they could not reject the unit root using a modified version of the Cogley and Sargent (2002) model where stationarity restrictions had been removed.

8. In the estimation of the ESTAR model, the transition parameter, γ , is estimated by scaling it by the variance of the transition variable. This scaling is suggested for two reasons. One is to avoid problems in the convergence of the algorithm. Second, it makes it easier to compare speeds of adjustment. Tables also report p -values for the null $\gamma = 0$ using bootstrap simulation.

TABLE 1a
RESULTS FOR ESTIMATED ESTAR MODEL

Estimated model: $y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-4}-a)^2} + \epsilon_t$						
a	β_1	β_2	β_3	γ	s	R^2
PCE inflation 1947Q1–1982Q4						
0.011 (0.00)	0.58 (0.09)	0.25 (0.11)	$1 - \beta_1 - \beta_2$	0.069 (0.028) [0.051]	0.0046	0.67
GARCH: Diagnostics:		$\varphi = 0.16$ $JB = 0.01$	$\phi = 0.78$ $Q(1) = 0.96$ $A(1) = 0.72$		$Q(4) = 0.85$ $A(4) = 0.68$	
PCE inflation 1983Q1–2004Q4						
0.0069 (0.00)	0.46 (0.11)	0.22 (0.11)	$1 - \beta_1 - \beta_2$	0.148 (0.074) [0.095]	0.0024	0.38
Diagnostics:		$JB = 0.33$	$Q(1) = 0.46$ $A(1) = 0.39$		$Q(4) = 0.22$ $A(4) = 0.58$	

NOTES: Figures in parentheses are the Newey–West standard errors. s denotes standard error of the regression. $Q(l)$, $A(l)$, and JB are the p -values of the Eitrheim and Teräsvirta (1996) LM test for autocorrelation in nonlinear series for l number of lags; the LM test for ARCH effects up to l lags, and the normality Jarque–Bera test, respectively. Figures in brackets represent the p -value of the γ parameter obtained through bootstrap simulation. GARCH(1, 1) takes the following form: $\sigma_t^2 = \theta + \varphi\epsilon_{t-1}^2 + \phi\sigma_{t-1}^2$.

TABLE 1b
ESTAR ESTIMATES FOR DIFFERENT REGIME POLICIES

Estimated model: $y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-4}-a)^2} + \epsilon_t$						
Period	a	β_1	β_2	γ	s	R^2
1960.1–1982.4	0.011 (0.00)	0.78 (0.11)	$1 - \beta_2$	0.066 [0.07]	0.0030	0.83
1966.1–1979.2	0.014 (0.00)	1		0.138 [0.06]	0.0032	0.70
1987.1–2004.4	0.007 (0.000)	0.65 (0.13)	$1 - \beta_2$	0.192 [0.08]	0.0024	0.41

NOTES: Brackets denote p -values using bootstrap simulation.

Table 1a are significant and inflation appears parsimoniously explained by an ESTAR process with three autoregressive lags. It is worth mentioning that the second period displays significantly lower value of α , the value at which the model displays local unit root behavior. Moreover, the second period exhibits significantly larger speed of adjustment of inflation toward α than the first period.

An alternative approach is to fit the ESTAR process for the whole period, allowing the intercept and the speed of adjustment to change by introduction of a dummy variable ($d82$). This takes the value of zero up to the fourth quarter of 1982 and unity afterward. To obtain critical values for the dummy variable coefficients, we employ the wild bootstrap that allows for heteroskedasticity (see, e.g., Davidson and Flachaire

TABLE 2
RESULTS FOR ESTIMATED ESTAR MODEL

Estimated model: $y_t = a + a * d82 + [B(L)(y_{t-4} - a - a * d82)e^{(-\gamma - \gamma * d82)(y_{t-4} - a - a * d82)^2}$								
a	a^*	β_1	β_2	β_3	γ	γ^*	s	R^2
U.S. PCE inflation 1953Q1–2004Q4								
0.011 (0.001)	-0.004 (0.0017) [0.10]	0.55 (0.08)	0.22 (0.08)	$1 - \beta_1 - \beta_2$	0.036 (0.015) [0.08]	0.54 (0.30) [0.11]	0.003	0.78
Diagnostics:		$Q(1) = 0.61$ $A(1) = 0.87$		$Q(4) = 0.25$ $A(4) = 0.13$		$JB = 0.01$		

NOTES: Figures in brackets represent the p -value of the t -statistics obtained through wild bootstrap simulation.

2008, Gonçalves and Kilian 2004, 2007).⁹ The result displayed in Table 2, for the sample period where the dummies are most significant, is consistent with the results reported in Table 1, confirming the global stationarity of the inflation series with local unit root behavior around different α values and different speed of response to shocks in the two periods.

1.3 Nonlinear Impulse Response Functions

To analyze the properties of the model with respect to local and global mean reversion, we examine the speed of mean reversion of the ESTAR model to shocks. To calculate the half-lives of inflation deviations ($y_t - a$) within the nonlinear framework, we need to obtain the generalized impulse response function (GIRF) for nonlinear models introduced by Koop, Pesaran, and Potter (1996). They differ from the linear response functions in that they depend on initial conditions, on the size and sign of the current shock, and on the future shocks as well. Appendix B describes in detail how the GIRFs are computed. In the case of nonlinear models, monotonicity in the impulse response need not hold and shock absorption becomes slower as the shock becomes smaller. Hence, we calculate the x -life of shocks for $(1 - x) = 0.50$, and 0.75 where $(1 - x)$ corresponds to the fraction of the initial shock that has been absorbed.

9. Employing each time the actual residuals from the model reported in Table 2, we create a new series of residuals based on these estimated residuals as $u_t^b = \hat{u}_t \epsilon_i$, where ϵ_i is drawn from the two-point distribution: $\epsilon_i = 1$ with probability $p = 0.5$; $\epsilon_i = -1$ with probability $p = 0.5$. The ϵ_i are mutually independent drawings from a distribution independent of the original data. The distribution has the properties that $E(\epsilon_i) = 0$, $E(\epsilon_i^2) = 1$, $E(\epsilon_i^3) = 0$, and $E(\epsilon_i^4) = 1$. As a consequence, any heteroskedasticity and non-normality due to the fourth moment in the estimated residuals, \hat{u}_t , is preserved in the created residuals, u_t^b . We then simulate the ESTAR model in Table 2, 10,000 times with the coefficients on the dummy variables set to zero, using residuals u_t^b , $i = 1, 2, \dots, 10,000$ and the actual initial values of y_{t-1} , y_{t-2} as starting values. We then estimate the ESTAR model with the dummy variables included to obtain the critical values. Analysis by Gonçalves and Kilian (2004) is suggestive, in a slightly different context, that the wild bootstrap will perform as well as the conventional bootstrap, which is based on resampling of residuals with replacement, even when the errors are homoskedastic. The converse is not true.

TABLE 3
GENERALIZED IMPULSE RESPONSE FUNCTION

Shock size	Absorption ($1 - x$)	Time period	
		1947–1982	1983–2004
$k = 1$	50%	5 (2.18)	3 (1.49)
	75%	12 (4.14)	7 (1.88)
$k = 3$	50%	5 (1.71)	3 (1.44)
	75%	11 (4.12)	6 (1.48)
$k = 5$	50%	3 (0.97)	1 (1.24)
	75%	7 (3.82)	4 (0.87)

NOTE: Numbers in parentheses denote the standard deviation of the GIRF.

Table 3 shows the results for the mean and standard deviation of the GIRFs in both subsamples. Two points are worth mentioning. First, the inflation series displays a nonlinear pattern. In particular, large shocks tend to be absorbed faster than small shocks.¹⁰ This fact can also be observed in Figures 1 and 2 where the mean impulse response functions plus and minus two standard deviations are depicted for both periods.¹¹ Second, inflation was significantly more persistent in the first period than in the second period.

These results emphasize the contrast between the global stationarity of the ESTAR process and the nonstationarity feature of integrated models such as random walk or IMA(1, 1) models where the GIRF would not die out after one and two periods, respectively.

2. FORECASTING

In this section, we examine the forecasting performance of four alternative univariate models of inflation, namely, a linear AR process, a random walk, an IMA/UC, and the ESTAR model reported earlier.¹² In all cases, the computation of the point

10. We point out that even though in many cases the GIRFs obtained within a given period for different shock sizes are not significantly different from each other, the nonlinear pattern applies in the sense that if we hit the series with smaller or larger shocks than the ones reported, then more significant differences would be obtained. Results are available upon request.

11. Note that due to the fact that the standard error differs across subsamples, the size of the shock also differs. This can be clearly seen in Figure 3 where the mean GIRFs for both periods are plotted on a common graph. Appendix B also discusses the comparison of shocks across subsamples.

12. See Stock and Watson (2007) for an analysis of the forecasting performance of univariate inflation processes relative to multivariate ones.

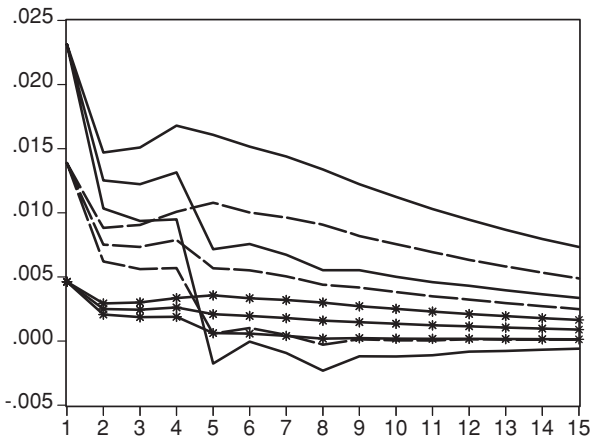


Fig. 1. Mean, Plus and Minus Two Standard Deviations of the GIRF: First Period, 1947–1982.

NOTES: Solid line: $k = 5$; dotted line: $k = 3$; star line: $k = 1$.

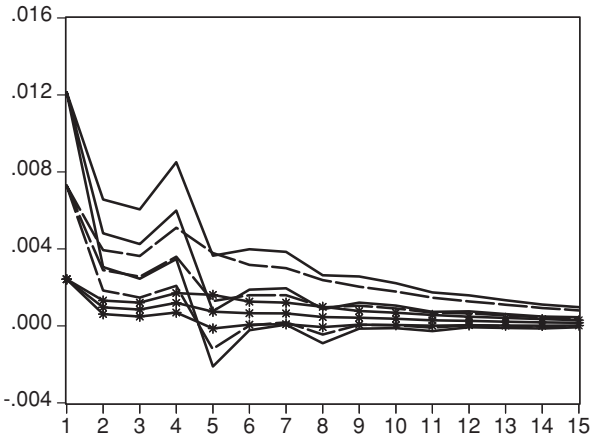


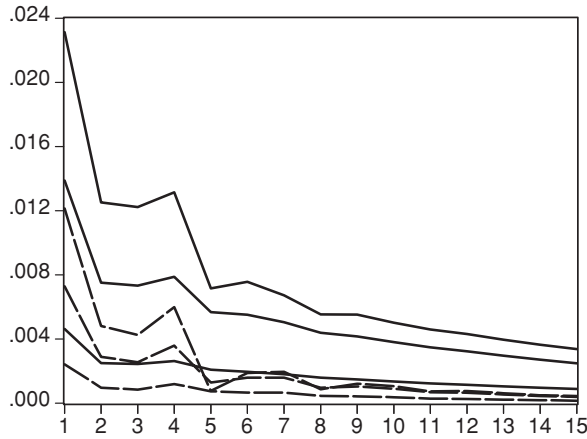
Fig. 2. Mean, Plus and Minus Two Standard Deviations of the GIRF: Second Period, 1983–2004.

NOTES: Solid line: $k = 5$; dotted line: $k = 3$; star line: $k = 1$.

forecast involves models with estimated parameters and, in many cases, alternative models nest each other.¹³ In particular, the ESTAR model can potentially nest either the random walk and the linear AR model.¹⁴ We therefore use the Clark and McCracken (2005) test for multistep forecast that normalizes the test statistic by the

13. Clements et al. (2003) point out the superiority of the DM statistic over density or interval forecasts to discriminate nonlinear models relative to linear ones.

14. Note that as we consider the same number of AR terms in the ESTAR and the linear AR model, the former model also nests the latter one.

FIG. 3. Mean GIRFs for Shock Size $k = 1, 3, 5$.

NOTES: Solid line: first Period, 1947–1982; dotted line: second period, 1983–2004.

error variance of the unrestricted model, in our case, the ESTAR. We construct the critical values using the bootstrap approach of Kilian (1999) and Kilian and Taylor (2003) as we cannot impose the assumption of i.i.d. errors.¹⁵ To compare the ESTAR model with the IMA/UC, we use the statistic developed by West (1996) and further discussed in McCracken (2004) given that the models are nonnested but there is parameter uncertainty.¹⁶

We undertake the forecasting exercise for the two separate subperiods over up to 2 years ahead ($h = 1, \dots, 8$). For initial observations, we use the period up to 1972Q4 in the first period and from 1983Q1 to 1994Q4 in the second period so we leave 10 years of out-of-sample in each case. The forecast is done recursively; i.e., as a new observation is added to the sample we reestimate the model and compute the forecast for h periods ahead. Table 4 reports the p -values for the null hypothesis test that the alternative ESTAR model does not have significantly different forecast accuracy, or mean square prediction error, relative to the three alternative models. The results for the first period show that the ESTAR model clearly outperforms the linear AR for horizons greater than 1 year, and the IMA at horizon 3. However, the ESTAR is inferior to the random walk up to three periods ahead, the IMA model one period ahead and not significantly different at other horizons.

15. The methodology consists on estimating each of the alternative models using the actual data and computing the statistics. To obtain the critical values, we generate a DGP using the restricted model (random walk or linear AR) with bootstrapped residuals. In every replication we estimate both the restricted and the unrestricted (ESTAR) model and compute the statistic. This is repeated 9,999 times to obtain the distribution of the test. The nonlinear forecasts are obtained from bootstrap simulation as is required for multiperiod nonlinear forecasts (see, e.g., Franses and van Dijk 2000). See Inoue and Kilian (2004) for a review of predictability tests.

16. The loss function we use for the forecasting evaluation is the mean square prediction error and therefore asymptotic irrelevance holds in this case, making the computation of the statistic simpler.

TABLE 4
FORECASTING PERFORMANCE OF NONLINEAR MODEL RELATIVE TO LINEAR

Period	Model	Horizon							
		1	2	3	4	5	6	7	8
1947– 1982	<i>RW – E</i>	0.98	0.95	0.97	0.82	0.66	0.60	0.52	0.55
	<i>Lin – E</i>	0.18	0.15	0.20	0.24	0.10	0.10	0.08	0.12
	<i>IMA – E</i>	0.99	0.66	0.09	0.98	0.74	0.17	0.52	0.71
1983– 2004	<i>RW – E</i>	0.00	0.01	0.27	0.10	0.37	0.28	0.35	0.23
	<i>Lin – E</i>	0.65	0.75	0.86	0.78	0.57	0.74	0.62	0.55
	<i>IMA – E</i>	0.94	0.92	0.78	0.75	0.70	0.53	0.51	0.46

NOTES: Numbers in table represent *p*-values. *RW – E* denotes the random walk model relative to ESTAR model, *Lin – E* denotes the linear model model relative to the ESTAR. *IMA – E* denotes the IMA model relative to ESTAR. Numbers in bold denote *p*-value equal or less than 10%.

For the second period, the ESTAR model outperforms the random walk over horizons 1, 2, and 4 but is inferior to the IMA model for horizons 1 and 2, and is not significantly better than the linear AR model. Overall, for our samples, the IMA model appears to provide superior forecasts to the ESTAR model over very short horizons.

3. CONCLUSIONS

In this paper, we have sought to examine a particularly unpalatable feature of inflation dynamics in the U.S., namely, its unit root property. The opportunistic approach to disinflation set out by Orphanides and Wilcox (2002) and Aksoy et al. (2006), and the risk management paradigm of central banks specified in Kilian and Manganelli (2007, 2008) provide a general framework that allows inflation to move within a band and can motivate the nonlinear ESTAR model of inflation. In this context, inflation follows a globally stationary process that could locally be nonstationary. We undertake a comprehensive array of statistical tests to show that ESTAR models parsimoniously capture the dynamic behavior of U.S. inflation in the postwar period. Our results show that while inflation is a near unit root process when close to equilibrium, it is globally mean reverting. This property is, *a priori*, surely more appealing from an economic perspective than the unit root alternative. Moreover, the implied dynamics, as derived from the impulse response functions, indicate distinctive speeds of adjustment between the generally accepted policy regimes. Overall, the results deliver adjustment speeds that are much faster and plausible than is implied in the extant literature.

From a forecasting perspective the results are more mixed. For our data samples the ESTAR model outperforms statistically, from a mean square forecast error perspective, the linear AR at horizons over a year in the first period, but the forecasts from these models are not significantly different from each other in the second period.

The random walk produces superior forecasts over short horizons to the ESTAR in the first period but is inferior to the ESTAR over short horizons in the second period. The IMA/UC model produces superior forecasts to the ESTAR model over horizons of one or two quarters in both periods and therefore appears superior for forecasting at short horizons. Nevertheless, given that the ESTAR model is better theoretically motivated and has desirable economic long-run properties, it appears to be a useful model for the analysis of inflation dynamics.

APPENDIX A: LINEARITY AND UNIT ROOT TESTS

We first consider the Escribano and Jorda (1999) (EJ hereafter) linearity LM test. The null hypothesis in this test (H_0^1) is that y_t follows a stationary linear process. The computation of the test is carried out using the F version which is an asymptotic Wald test. If linearity is rejected, we follow the EJ procedure to discriminate between the ESTAR and LSTAR nonlinear models.

In the case that the errors display heteroskedasticity, Lundbergh and Teräsvirta (1998) conclude that the robust version of the LM linearity tests appears to have very low power. Pavlidis, Paya, and Peel (2009) show that in the case of the EJ test the fixed design wild bootstrap appears to improve the test in terms of its size under heteroskedastic errors. Gonçalves and Kilian (2004) analyse the asymptotic validity of the wild bootstrap within stationary autoregressions with conditional heteroskedastic errors. Their results can be generalized to the unit root case following Inoue and Kilian (2002). We therefore apply the Pavlidis, Paya, and Peel bootstrap version of the test, and for the null of a linear stationary process (H_0^1) in the U.S. inflation series we obtain p -values of 0.000 and 0.012 for the first and second periods, respectively. The selected values for the parameters (p, d) are (3, 4) as they correspond to the most parsimonious model that rejects (H_0^1) in both periods. We also note that a delay of four quarters is *a priori* reasonable given that the nonlinearity is attributed to policy actions, and a four-quarter moving average of inflation is employed in some empirical estimates of Taylor rules. These are the parameter values (p, d) we will consequently use for the rest of the article. Moreover the minimum p -value corresponds to the F -test of the ESTAR alternative in both periods, and consequently, it is possible to reject the null of linear stationary process in favor of a nonlinear stationary ESTAR model.

An alternative linearity testing procedure would be given theoretical priors to have a linear unit root inflation as the null hypothesis. Consequently, we also undertake the tests of Kapetanios, Shin, and Snell (2003) (KSS hereafter) where the null of a linear unit root process is tested against the alternative of a globally stationary nonlinear ESTAR model. We obtain values for the KSS test of -4.36 and -2.96 for the two subperiods. These values are significant using the conventional critical values provided in KSS, therefore suggesting we can reject the null of a unit root in inflation in favor of the ESTAR process.

To make certain that the implementation of the KSS test is robust within our framework we carry out a Monte Carlo exercise. In particular, we generate the true DGP as the IMA model of Stock and Watson calibrated with the values of each subsample. We use the same sample size as the actual data, which is 144 observations for the first period and 88 for the second one, and simulate 9,999 data samples for each subperiod. We then apply the KSS test to this simulated data for each subperiod to obtain the new critical values. The values obtained above for the real data would correspond with p -values of 0.058 and 0.087 for each period, respectively. Consequently, the KSS test points to a rejection of the null of a linear unit root in favor of an ESTAR process.

The third linearity test we perform is the one developed in Harvey and Leybourne (2007) (HL hereafter). They test the null hypothesis of a linear process, which could be either stationary or nonstationary, since their statistic is consistent against either form. The values we obtain for the first and second periods are 21.15 and 3.97, respectively. Linearity is clearly rejected in the first period but not in the second one. However, if, for the second period, we include a third power in the transition variable of the test regression using the same rational as EJ, that is, to increase the power of the test against the ESTAR, the test rejects the null hypothesis with a p -value of 0.07.

A second step of the HL test is to determine the stationarity or nonstationarity of the processes using the Harris, McCabe, and Leybourne (2003) test statistic. In our case stationarity could not be rejected. Given the existence of a discrepancy between the KSS and the HL tests for the second period, we check the power of both statistics under the alternative of an ESTAR process with a range of parameter values similar to the ones obtained in the estimation provided in the next section. The KSS test appears to be more powerful in this case as, according to Table 3 in KSS and Table 3 in HL, the power of the KSS and HL tests is 0.98 and 0.25, respectively.

APPENDIX B: GENERALIZED IMPULSE RESPONSE FUNCTIONS (GIRF)

The GIRF is defined as the average difference between two realizations of the stochastic process $\{y_{t+h}\}$ that start with identical histories up to time $t-1$ (initial conditions) but one realization is “hit” by a shock at time t while for the other one is not

$$\text{GIRF}_h(h, \delta, \omega_{t-1}) = E(y_{t+h} | u_t = \delta, \omega_{t-1}) - E(y_{t+h} | u_t = 0, \omega_{t-1}), \quad (\text{B1})$$

where $h = 1, 2, \dots$, denotes horizon; $u_t = \delta$ is an arbitrary shock occurring at time t ; and ω_{t-1} defines the history set of y_t . The value of (B1) has to be approximated using stochastic simulation since it is not possible to obtain an analytic expression for the conditional expectation involved in (B1) for horizons larger than one (see Koop, Pesaran, and Potter 1996). A comprehensive analysis of impulse responses and estimating procedures can be seen in Kilian and Zha (2002). For each history,

we construct 5,000 replications of the sample paths $\hat{y}_0^*, \dots, \hat{y}_h^*$ based on $u_t = \delta$ and $u_t = 0$ by randomly drawn residuals as noise for $h \geq 1$. The difference of these paths is averaged across the 5,000 replications and it is stored. To obtain the final value for (B1), we average across all histories.

For a particular value of inflation at time t , the series is hit with a shock of size δ . The shock size is usually determined in terms of the residual standard deviation ($\hat{\sigma}_u$) of the model, such that $\delta = k\hat{\sigma}_u$. In this way, one can compare shocks absorption for a given value of k but for models with different standard errors. Moreover, it is also possible to convert it to a common measure in terms of the level of the dependent variable. In our case, the residual standard deviation in the first period is $\hat{\sigma}_{1,u} = 0.0046$, which corresponds roughly to an additive 2% per annum shock on the level of inflation at quarter t . In the first subsample, the largest change in inflation on a given quarter took place in the early 50s and was equal to $0.024 (\simeq 5\hat{\sigma}_{1,u})$, or roughly 10% in annual terms. However, in the second period, $\hat{\sigma}_{2,u} = 0.0024$ which corresponds to a 1% per annum shock to inflation level in a particular quarter. The largest change in inflation in the second subsample equals $0.007 (\simeq 3\hat{\sigma}_{2,u})$ and took place in the 80s. We therefore consider the following set of values for $k = 1, 3, 5$. The particular choice of k s allows us to compare and contrast the persistence of small, and large shocks within and across periods.

To measure the x -life or x -absorption time, we follow van Dijk, Franses, and Boswijk (2007) and compute

$$N(x, \delta_t, \omega_{t-1}) = \sum_{m=0}^{\infty} \left(1 - \prod_{h=m}^{\infty} I(x, h, \delta_t, \omega_{t-1}) \right),$$

where $0 \leq x \leq 1$ and $I(x, h, \delta_t, \omega_{t-1})$ is the indicator function, which takes the value of 1 if at least a fraction $1 - x$ of the difference between the initial and ultimate effects of δ_t has been absorbed after h periods and 0 otherwise. The indicator function is defined as

$$I(x, h, \delta_t, \omega_{t-1}) = I[|\text{GIRF}(h, \delta_t, \omega_{t-1}) - \text{GIRF}^{\infty}(\delta_t, \omega_{t-1})| \leq \pi |\delta_t - \text{GIRF}^{\infty}(\delta_t, \omega_{t-1})|].$$

Note that the above definition of x -life differs from the definition usually adopted in the literature, which is the shortest horizon at which at least a fraction $1 - x$ of the initial effect, δ_t , has been absorbed. This is an appealing feature since monotonicity is not granted. That is, $I(x, h, \delta_t, \omega_{t-1}) = 1$ does not necessarily imply $I(x, h + j, \delta_t, \omega_{t-1}) = 1, \forall j > 0$.

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